

CORRECTION DEVOIR SURVEILLÉ N°5 (40MIN)

Factoriser si nécessaire puis résoudre les équations suivantes dans $\mathbb{R}$:

1. $(3x + 1)(2x - 9) = 0$

$$\begin{aligned}(3x + 1)(2x - 9) &= 0 \\ \Leftrightarrow 3x + 1 &= 0 \quad \text{ou} \quad 2x - 9 = 0 \\ \Leftrightarrow 3x &= -1 \quad \text{ou} \quad 2x = 9 \\ \Leftrightarrow x &= -\frac{1}{3} \quad \text{ou} \quad x = \frac{9}{2}\end{aligned}$$

$$\mathcal{S} = \left\{ -\frac{1}{3}; \frac{9}{2} \right\}$$

2. $5x^2 - x = 0$

$$\begin{aligned}5x^2 - x &= 0 \\ \Leftrightarrow x(5x - 1) &= 0 \\ \Leftrightarrow x &= 0 \quad \text{ou} \quad 5x - 1 = 0 \\ \Leftrightarrow x &= 0 \quad \text{ou} \quad 5x = 1 \\ \Leftrightarrow x &= 0 \quad \text{ou} \quad x = \frac{1}{5}\end{aligned}$$

$$\mathcal{S} = \left\{ 0; \frac{1}{5} \right\}$$

3. $(2x + 6)(4x + 1) + (x - 5)(2x + 6) = 0$

$$\begin{aligned}(2x + 6)(4x + 1) + (x - 5)(2x + 6) &= 0 \\ \Leftrightarrow (2x + 6)[(4x + 1) + (x - 5)] &= 0 \\ \Leftrightarrow (2x + 6)(5x - 4) &= 0 \\ \Leftrightarrow 2x + 6 &= 0 \quad \text{ou} \quad 5x - 4 = 0 \\ \Leftrightarrow 2x &= -6 \quad \text{ou} \quad 5x = 4 \\ \Leftrightarrow x &= -3 \quad \text{ou} \quad x = \frac{4}{5}\end{aligned}$$

$$\mathcal{S} = \left\{ -3; \frac{4}{5} \right\}$$

4. $4x^2 - 36 = 0$

$$\begin{aligned}4x^2 - 36 &= 0 \\ \Leftrightarrow (2x)^2 - 6^2 &= 0 \\ \Leftrightarrow (2x - 6)(2x + 6) &= 0 \\ \Leftrightarrow 2x - 6 &= 0 \quad \text{ou} \quad 2x + 6 = 0 \\ \Leftrightarrow 2x &= 6 \quad \text{ou} \quad 2x = -6 \\ \Leftrightarrow x &= 3 \quad \text{ou} \quad x = -3\end{aligned}$$

$$\mathcal{S} = \{-3; 3\}$$

5. $(x+3)^2 - (-2x+5)^2 = 0$

$$\begin{aligned}
 & (x+3)^2 - (-2x+5)^2 = 0 \\
 \iff & [(x+3) - (-2x+5)][(x+3) + (-2x+5)] = 0 \\
 \iff & (x+3+2x-5)(x+3-2x+5) = 0 \\
 \iff & (3x-2)(-x+8) = 0 \\
 \iff & 3x-2 = 0 \quad \text{ou} \quad -x+8 = 0 \\
 \iff & 3x = 2 \quad \text{ou} \quad -x = -8 \\
 \iff & x = \frac{2}{3} \quad \text{ou} \quad x = 8
 \end{aligned}$$

$$\mathcal{S} = \left\{ \frac{2}{3}; 8 \right\}$$

6. $(x+2)(4x+1) - (x+2)(x-1) = 0$

$$\begin{aligned}
 & (x+2)(4x+1) - (x+2)(x-1) = 0 \\
 \iff & (x+2)[(4x+1) - (x-1)] = 0 \\
 \iff & (x+2)(4x+1-x+1) = 0 \\
 \iff & (x+2)(3x+2) = 0 \\
 \iff & x+2 = 0 \quad \text{ou} \quad 3x+2 = 0 \\
 \iff & x = -2 \quad \text{ou} \quad 3x = -2 \\
 \iff & x = -2 \quad \text{ou} \quad x = -\frac{2}{3}
 \end{aligned}$$

$$\mathcal{S} = \left\{ -2; -\frac{2}{3} \right\}$$

7. $(3+x)^2 - 2(3+x)(x-1) = 0$

$$\begin{aligned}
 & (3+x)^2 - 2(3+x)(x-1) = 0 \\
 \iff & (3+x) \times (3+x) - 2(x-1) \times (3+x) = 0 \\
 \iff & (3+x)[(3+x) - 2(x-1)] = 0 \\
 \iff & (3+x)(3+x-2x+2) = 0 \\
 \iff & (3+x)(-x+5) = 0 \\
 \iff & 3+x = 0 \quad \text{ou} \quad -x+5 = 0 \\
 \iff & x = -3 \quad \text{ou} \quad x = 5
 \end{aligned}$$

$$\mathcal{S} = \{-3; 5\}$$

8. $2x^2 + 8x + 8 = 0$

$$\begin{aligned}
 & 2x^2 + 8x + 8 = 0 \\
 \iff & x^2 + 4x + 4 = 0 \\
 \iff & (x+2)^2 = 0 \\
 \iff & x+2 = 0 \\
 \iff & x = -2
 \end{aligned}$$

$$\mathcal{S} = \{-2\}$$

9. $3x^2 - 4x + 2 = -6x^2 + 2x + 1$

$$\begin{aligned}
 & 3x^2 - 4x + 2 = -6x^2 + 2x + 1 \\
 \iff & 3x^2 - 4x + 2 + 6x^2 - 2x - 1 = 0 \\
 \iff & 9x^2 - 6x + 1 = 0 \\
 \iff & (3x - 1)^2 = 0 \\
 \iff & 3x - 1 = 0 \\
 \iff & 3x = 1 \\
 \iff & x = \frac{1}{3}
 \end{aligned}$$

$$\mathcal{S} = \left\{ \frac{1}{3} \right\}$$

10. **Bonus :** En utilisant la deuxième identité remarquable puis la troisième identité remarquable, factoriser $x^2 - 6x - 16$.

$$\begin{aligned}
 x^2 - 6x - 16 &= \boxed{x^2 - 6x + 9} - 9 - 16 \\
 &= (x - 3)^2 - 25 \\
 &= (x - 3)^2 - 5^2 \\
 &= (x - 3 - 5)(x - 3 + 5) \\
 &= (x - 8)(x + 2)
 \end{aligned}$$

on fait apparaître la 2^{ème} identité remarquable
car $x^2 - 6x + 9 = (x - 3)^2$

grâce à la 3^{ème} identité remarquable